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Research article

Comparison: Binomial model and Black Scholes model

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Abstract: The Binomial Model and the Black Scholes Model are the popular methods that are used to solve the option pricing problems. Binomial Model is a simple statistical method and Black Scholes model requires a solution of a stochastic differential equation. Pricing of European call and a put option is a very difficult method used by actuaries. The main goal of this study is to differentiate the Binominal model and the Black Scholes model by using two statistical model - t-test and Tukey model at one period. Finally, the result showed that there is no significant difference between the means of the European options by using the above two models.

Keywords: European options; Binominal model; Black Scholes model; t-test; Tukey model
JEL classification numbers: G12

Abbreviation: BM: Binomial Model; BSM: Black Scholes Model; CBM: Calibrating Binomial Model; GBM: Geometric Brownian motion

1. Introduction

The idea of an option is not new. Options were traded in Ancient Romans against outdoing cargoes from their seaport. The options are the main dynamic segment of the security market, since the origin of the Chicago Board option exchange (CBOF) in April 1997. It is the largest options exchange in the world with more than one million contracts per day (Boyle, 1977). The option is a type of a derivative. It is defined as "It gives the holder the rights but not obligation to buy in case of a call option or sell in the case put option of an underlying asset at a fixed price (exercise or strike price) on the maturity (expiry date)". Options are used for hedging and speculation. Hedging is nothing it is simply a process to reduce the future loss and speculation is to make a position in the market (Hull, 2006). Option theory has a long history, but it was not until Black and Scholes presented the first complete equilibrium option pricing model in the year 1973. In the same year, Robert Merton explains the BSM in different ways. From 1973 when BSM came in the market it has been widely used by traders to determine the price of the option (Oduro, 2012). Another option model existed in the market namely BM developed by Cox-Ross-Rubinstein in 1979 (Cox et al., 1979). BM is a simple and easy to understand. Jump diffusion model suggested by Merton in 1973. The Monte Carlo simulation method was developed by Boyle in the year 1977.

Pricing of options (call/put) is one of the most important aspects of trading the derivatives. The two models BSM and BM are famous in the financial market. In 1973 Fisher Black and Myron Scholes, they develop an option pricing model called BSM for European style options without dividends which allowed investors to approximate a price. Previous models by Sprenkle in 1961, Ayes in 1963, Boness in 1964 and others had determined the value of the option in terms of warrants (Black and Scholes, 1973). A warrant is a contract that confirms the right, but not obligation to buy or sell. It is issued by a firm, not a stock exchange. The assumption of the BSM is that the underlying assets follow a lognormal distribution before BSM researchers such as Mandelbrot in 1963, Fama in 1965 and Blattberg and Gonedes in 1974 suggesting that it is following student t distribution. In 1973, Robert Merton modified the BSM to account for dividends and variables interest rate. In 1976, Jonathan Ingersoll gave an assumption for BSM that is there is no tax or transaction cost. John Cox and Stephen Ross in 1976 showed that underlying assets do not have to move continuously, but the price of the underlying goes from one price to another.

Feng and Kwan (2012), investigated that the BM which eventually converges to well-known BSM as the number of Binomial period increase. In the year, 2012 the Oduro and Dedu showed the relationship between the BM and the BSM with the help of MS VBA. In the year 2014, Lazarova *et al*. investigated that the two modes BM and the BSM do not affect the crucial idea that underlies the option of each model. In the year 2017, Dar and Anuradha showed that the BM and the BSM have the negligible difference between them with the help of MATLAB by simulations.

Different parameters of data are estimated by using MS-Excel. Initially, the basic terminologies of options are studied like some definitions of derivatives, types of options, option styles are mentioned. In BM we use lattices to estimate the value of the option (call/put) at the initial time (t=0) that provides a payoff for each call and put options at different strike prices that we generated by using excel function at an expiry/mature date based on the non-dividend paying shares and without premium. Also, we calculated the price of each option by applying a direct formula known as Black-Sholes formula. Finally, the results we got that BSM is approximately equal to BM at one period with the help of statistical t-test and Tukey method. MINITAB and SPSS have been used as a necessary tool to calculate all the important results.

1.1. Objectives

To understand how BM and BSM are approximately equal at one period by using t-test and Turkey method or in other words, there is no mean difference between the BM and BSM at one period.

1.2. Methodology

Data have taken from State Bank of India on monthly from 1st January 2010 to 31st December 2010 monthly basis from www.finance.yahoo.com. First of all, we calculated the price of a European option at one period by using the two famous pricing models BM and BSM. Later we compare both the models in order to check whether there are any mean differences between the BM and BSM at one period with the help of statistical tools - t-test and Turkey method.

2. Terminology

2.1. Derivative

A derivative is a contract/security which gives us promise to make payment on some future date. It is a financial tool whose value depends on some underlying assets. The underlying assets are shares, bonds, index, interest rate, currency, commodity (gold, wheat etc).

2.2. Options

It is a contract between individuals or firms in which one party is ready to buy and another party is ready to sell. "An option is a contract that gives the owner the right to buy or sell any underlying asset at a fixed price (the predetermined price of an underlying asset) on the future date".

There are two types of options:

2.2.1. Call option

"It gives the rights to the holder of an option but not obligation to buy an underlying asset at a strike price on the future date".

2.2.2. Put option:

"It gives the rights to the holder of an option but not obligation to sell an underlying asset at a strike price on the future date".

2.3. Option style

There are two styles of the options which are defined below:

2.3.1. American options

"American option is a contract that gives the holder right to buy or sell any underlying asset at a strike price on or before the expiry date". The owner can exercise this option anytime before the expiry date.

2.3.2. European options

"European Option is a contract that gives the right to buy or sell any underlying asset at a fixed strike price on the exercise date". The European option only exercises at maturity date not before.

2.4. Notations

Below are some notations that are using in this paper:

- t - The current time;
- S_t - The underlying share price at time t;
- K - The strike/exercise price;
- T - The option exercise date;
- r is the interest rate;

2.5. Payoff

The money realised by the holder of an option at the end of its life (Hull, 2006). The profit made by the holder at the maturity date is known as a payoff.

2.5.1. Payoff of European call option

Let us consider a call option with S_t is the price of the underlying at time t and fixed strike price K.

At expiry time T we have two different cases:

(a) "At expiry time T, if the price of strike price K is less than the price of underlying asset S_T, then the call option is exercised i.e. the holder buys the underlying asset at price K and immediately sell it in the market at price S_T, the holder realize the profit".

$$Payoff = S_T - k$$

(b) "At expiry time T, if the price of strike price K is greater than the price of underlying asset S_T, then the call option is not exercised". The option expires worthless with $payoff = 0$.

Combine the above two cases, at time T. The value of a call option is given by a function

$$C_u = \max(S_T - k, 0)$$

Figure 1 shows payoff of a European call option without premium.

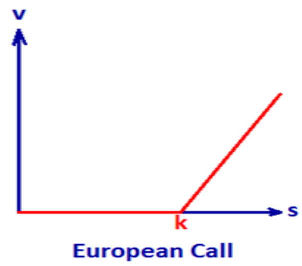

Figure 1. Payoffs of a European call option.

Note: In a call option, the holders expect that the price of the underlying asset will increase in future.

2.5.2. Payoff of European put option

Let us consider a put option with S_t is the price of the underlying at time t and fixed strike price K.

At expiry time T we have two different cases:

(a) "At the expiry date T, if the strike price K is greater than the underlying asset S_T, then the put option is exercised, i.e. the holder buys the underlying asset from the market price S_T and sell it to the writer at price K". The holder realizes the profit of $K - S_T$.

(b) "At expiry time T, if the strike price K is less than the price of underlying asset S_T, than the put option is not exercised". The option expires worthless with payoff = 0.

Combine the above two cases, at time T. The value of a put option is given by a function

$$c_d = \max(K - S_T, 0)$$

Figure 2 shows payoff of a European put option.

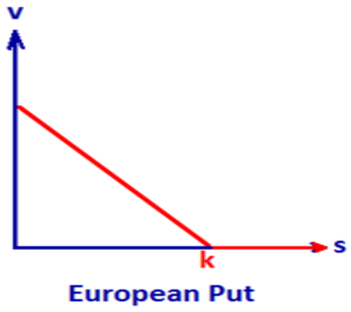

Figure 2. Payoff of a European put option.

Note: The holder expects that the price of the underlying asset will decrease.

2.6. Hedging

"It reduces the future risk in the underlying by entering into a derivative contract whose value moves in the opposite direction to the underlying position and cancels part or all of it out (Hull, 2006)".

2.7. Speculation

"It is a sort of winning or losing the amount of money I .e; gambling, in this a person even gains a big margin of money or loses a lot of amounts (Hull, 2006)".

2.8. Arbitrage

"Arbitrage means risk-free trading profit"; it is something described as a free breakfast.

Arbitrage opportunity exists if

(i) If we make an immediate profit with a probability of loss is zero.

(ii) With a zero cost initial investment if we can get money in future.

2.9. No arbitrage

"No-arbitrage means when arbitrage opportunity does not exist in the market".

3. Binominal model

The BM is also known as Cox-Ross-Rubinstein (CRR) model because J.C Cox, S.A Ross and M. Rubinstein developed BM in the year 1979. Options play a necessary role in the financial market as widely applied in financial derivatives (Liu and Chen, 2009).BM is a model to estimate the value of an option at time $t = 0$ (at the beginning) that provides a payoff at a future date based on the value of a non-dividend paying shares at a future date. The BM is based on the assumption that there is no arbitrage opportunity exist in the market. The assumption is that the stock price follows a random walk. In every step, it has a certain probability of moving up or down.

We consider a one-step BM. In this model, we start at time $t = 0$ when the underlying asset price is equal to S_t. Over a time period (say one) the underlying asset prices do one of the two things (Dar and Anuradha, 2017). It will be either

Jump upward with some probability say q

Jump downward with some probability say $1 - q$

At time $t = 1$ we have two possibilities

$$S_1 = \begin{cases} S_0 * u, if\ the\ underlying\ asset\ price\ goes\ up \\ S_0 * d, if\ the\ underlying\ asset\ price\ goes\ down \end{cases}$$

Note: u is always greater than 1 and d is less than 1.

Figure 3 represents the one step BM

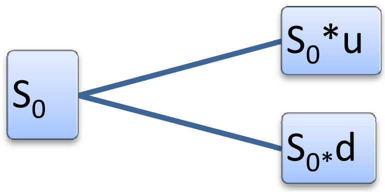

Figure 3. One step BM.

Figure 3 is one period model because we consider only time interval from 0 to 1, it is Binomial because there are only two possibilities either goes up $S_0 * u$ with probability q or down $S_0 * d = S_1$ with probability $1 - q$. The above model is referred as Binomial tree and each path from S_0 to $S_0 * u$ and from S_0 to $S_0 * d$ as a branch. The points at the end of the branch are known as nodes (Dar and Anuradha, 2017).

We can now see the implication of there being no arbitrage in this model. In order to avoid arbitrage, it is necessary that we must have $d < e^r < u$.

The derivative price (one period) at time $\boldsymbol{t = 0}$ is

$$V_0 = e^{-r}\left(C_u * q + C_d * (1 - q)\right) - - - (1)$$

Where $q = \frac{e^r - d}{u - d}$ and $1 - q = \frac{u - e^r}{u - d}$

Note that the no-arbitrage conditions $d < e^r < u$ ensures that $0 < q < 1$.

Proof of equation 1:

We consider a one-period binomial model as shown in figure 3; we have two possibilities at time $\mathrm{t} = 1$ i.e.

$$\mathrm{S_1} = \begin{cases} \mathrm{S_0 * u} & \text{if the underlying asset price goes up} \\ \mathrm{S_0 * d} & \text{if the underlying asset price goes down} \end{cases}$$

Here $\mathrm{u} > 1$ and $\mathrm{d} < 1$

In order to divert arbitrage, we must have $\mathrm{d} < \mathrm{e^r} < u$

Where,

- "r is the risk-free rate of interest"

Let at $\mathrm{t} = 0$, we hold a portfolio that consists of α unit of the underlying asset and β unit of cash. So the value of the portfolio at time $\mathrm{t} = 0$ is:

$$V_0 = \alpha * S_0 + \beta ----[2]$$

The value of the portfolio at time $t = 1$ will be

$$V_1 = \begin{cases} \alpha * S_0 * u + \beta e^r & \text{if the underlying asset price goes up} \\ \alpha * S_0 * d + \beta e^r & \text{if the underlying asset price goes down} \end{cases} ----[3]$$

Let the derivatives pay c_u if the prices of the underlying asset go up and c_d if the prices of the underlying asset go down.

Let us choose α and β so that

$$V_1 = \begin{cases} c_u & \text{if the underlying asset price goes up} \\ c_d & \text{if the underlying asset price goes down} \end{cases} ----[4]$$

So, from equation (3) and (4) we get

$$\alpha * S_0 * u + \beta e^r = c_u --- [5]$$

$$\alpha * S_0 * d + \beta e^r = c_d --- [6]$$

Subtract equation (5) and (6) we get

$$\alpha * S_0 * u - \alpha * S_0 * d = c_u - c_d$$

$$\alpha * S_0(u - d) = c_u - c_d$$

$$\alpha = \frac{c_u - c_d}{S_0(u - d)}$$

Put value of α in equation (6) we get

$$\beta = e^{-r}(\frac{c_d * u - c_u * d}{u - d})$$

Substitute the value of α and β in equation (2) we get

$$V_0 = e^{-r}\left\{c_u\left(\frac{e^r - d}{u - d}\right) + c_d\left(\frac{u - e^r}{u - d}\right)\right\}$$

$$V_0 = e^{-r}\left\{c_u\left(\frac{e^r - d}{u - d}\right) + c_d\left(1 - \frac{e^r - u}{u - d}\right)\right\}$$

$$V_0 = e^{-r}(c_u * q + c_d(1 - q))$$

Where Synthetic probability (q) is simple number defined as:

$$q = \frac{e^r - d}{u - d}$$

If we denote the payoff of the options at time $t = 1$ by a random variable by c_t, we can write equation (1) as:

$$V_0 = e^{-r}E_Q(c_t)$$

Where Q is a probability measure which gives probability q for upward and $1 - q$ for downward and q is simple a real number and it is known as synthetic probability.

3.1. Calibrating Binomial Model

In a CBM we have mean and variance implies by the BM correspond to the mean and variance of the lognormal distribution. For recombining BM, an additional condition that leads to a unique solution is:

$$u = \frac{1}{d}$$

It means that an up step and a down step would mean the stock price, after two steps, is the same as it is at time $t = 0$.

Here we are using an assumption that the underlying stock follows a Geometric Brownian Motion (GBM). I.e. the underlying stock price changes continuously through time according to the stochastic differential equation (SDE):

$$dS_t = S_t(\mu dt + \sigma dZ_t)$$

Recall that this is same as the lognormal model. Here we showed that if $dS_t = S_t(\mu dt + \sigma dZ_t)$, then $\frac{S_t}{S_0}$ has a lognormal distribution with parameters $(\alpha - 0.5 * \sigma^2)t$ and $\sigma^2 t$.

Under the risk-neutral law if we parameterise the lognormal distribution so that:

$$ln\{S_t/S_{t_0}\} \sim N\left[\left(r - \frac{\sigma^2}{2}\right)(t-0), \sigma^2(t-t_0)\right]$$

Then the conditions that must be met are:

$$E[S(t+\delta t)/S(t)] = e^{r\delta t} --- (a)$$

And

$$var[\ln\{S(t+\delta t)/S(t)\}] = \sigma^2 \delta t --- (b)$$

Where:

δt is the time interval of each step in the BM

$S(t)$ or S_t denotes the price of the underlying asset at time t.

Noting that:

$$E[S(t+\delta t)/S(t)] = qu + (1-q)d$$

It follows from the equation (a) that:

$$qu + (1-q)d = e^{r\delta t}$$

Solve the above equation

$$q = \frac{e^{r\delta t} - d}{u - d}$$

Using the equation b and assume that $u = \frac{1}{d}$

$$var[\ln\{S(t+\delta t)/S(t)\}] = q(\ln(\mathrm{u}))^2 + (1-\mathrm{q})(-\ln(\mathrm{u}))^2 - E[\ln\{S(t+\delta t)/S(t)\}]^2$$
$$= (\ln(u))^2 - E[\ln\{S(t+\delta t)/S(t)\}]^2 --- (c)$$

The last term involves terms of higher order than δt

i.e.

$$E[\ln\{S(t+\delta t))/S(t)\}]^2 = f\{(\delta t)^2\} --- (d)$$

When $\delta t \to 0$ the equation d tends to 0

So, if we ignore the above equation and equate the equation c to $\sigma^2 t$, we get

$$var[\ln\{S(t+\delta t)/S(t)\}] = (\ln(u))^2 = \sigma^2 t --- (e)$$

If we solve the equation e, we get the value of u

$$u = e^{\sigma\sqrt{\delta t}}$$

And w know than $d = \frac{1}{u}$, so

$$d = e^{-\sigma\sqrt{\delta t}}$$

4. Black Scholes model

The Black Scholes Merton model (sometimes also known as Black Scholes model) was developed in the year 1973 by the famous economists namely Fisher Black, Robert Merton and Myron Scholes. Because of this model R. Merton and M. Scholes awarded by the Nobel Prize in

1977. Nowadays it is a famous model for estimating the values of the options. It has some assumptions that are given are - the price of an underlying asset follow a GBM, Arbitrage opportunity does not exist in the market, there are no taxes, the risk-free rate of interest is constant and also the volatility (Hull and White, 1987). Essentially, BSM formula shows us how to find the price of an option contract (call and put option) can be determined by using a simple formula. The formula for European call "c" and put option "p" is:

$$c = S_t N(d_1) - Ke^{-rT} N(d_2) - - - - [7]$$

$$p = Ke^{-rT} N(-d_2) - S_t N(-d_1) - - - - [8]$$

Where,

$$d_1 = \frac{\ln\left(\frac{S_t}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$d_2 = \frac{\ln\left(\frac{S_t}{K}\right) + \left(r - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

"S_t is the present price of the price of an underlying asset, K is the strike price, r is the free risk rate interest, σ is the volatility of the stock and $N(*)$ is the cumulative distribution function for a standard normal distribution, $S_t N(d_1)$ is the present value of the underlying asset if the option is exercised and Ke^{-rT} is the present value of the strike price if the option is exercised (Lazarova et al., 2014)".

5. Estimating the parameters

We have taken the historical data of State Bank of India-NSE of one year starting from 1st Jan 2010 to 31st Dec 2010 to estimate the parameters. The risk-free rates of interest, volatility have been estimated from historical data.

5.1. Volatility or standard deviation

The underlying asset price S_t is observed only at fixed intervals. When we speak about the historical volatility of an underlying asset price, it means we actually mean historical volatility of return. It is a measure of variation of underlying asset price over a time period (Baxter and Rennie, 1996). Mathematically historical volatility is an approximate same as the standard deviation of the return.

$$\sigma = \sqrt{\sum_{t=1}^{n} \frac{(v_t - \overline{v_t})^2}{t-1}}$$

Where, $v_t = \ln\left(\frac{S_t}{S_{t-1}}\right)$

St is the stock price, $\overline{S_t}$ is the mean of the stock price and t is a number of observations.

After calculation we have

Standard deviation/Volatility $\sigma = 4.1317\%$

5.2 Calculation of parameters, d, u, and q

A non-dividend paying underlying assets has volatility $\sigma = 4.1317\%$. We can easily calculate the value of the parameters u, d and q by using the following formulae

- $u = e^{\sigma\sqrt{\delta t}}$
- $d = e^{-\sigma\sqrt{\delta t}}$
- $q = \frac{e^{r\delta t}}{u-d} - \frac{d}{u-d}$

Where σ (volatility) as is computed above, $r = -0.45513\%$ is risk-free interest or expected return, $\delta t = 1$ and e^r =0.995459041

After calculation we have

- $u = 1.042182$
- $d = 0.959525$
- $q = 0.434735$
- $1 - q = 0.565265$

We can use above information to check our assumption i.e. no arbitrage opportunity exist in the market or not. In order to avoid arbitrage we must have $d < e^r < u$.

The above information satisfies our inequality $d < e^r < u$. Hence there is no arbitrage opportunity exist in the market.

5.3 Value of call and put option in BM

In BM we use lattices to estimate the value of the option (call/put) at an initial time ($t = 0$) that provides a payoff for each call and put options at different strike prices that we generated by using excel function at a expiry/mature date based on the non-dividend paying shares and without premium. The below table 1 shows the value of a call and put option using the equation (1) at different strikes prices and we used ln ($close\ price$) of SBI as share price at time t=0.

Table 1. Value of call and put option in BM.

Date	S_0	$Max(S_1$-K,0)	$Max(K - S_1$,0)	$Max(S_2$-K,0)	$Max(K - S_2$,0)	call option/ V_0	put option/ V_0
01-01-2010	7.62881	0.00000	3.04939	0.00000	3.67997	0.00000	3.42137
01-02-2010	7.58797	1.90805	0.00000	1.28085	0.00000	1.56060	0.00000
02-03-2010	7.63926	0.00000	7.03850	0.00000	7.66994	0.00000	7.42917
01-04-2010	7.74097	4.06750	0.00000	3.42765	0.00000	3.72272	0.00000
03-05-2010	7.72701	0.00000	0.94705	0.00000	1.58575	0.00000	1.31405
01-06-2010	7.74153	4.06809	0.00000	3.42819	0.00000	3.72329	0.00000
01-07-2010	7.82521	0.15529	0.00000	0.00000	0.49152	0.06782	0.27911
02-08-2010	7.92530	0.00000	3.74039	0.00000	4.39548	0.00000	4.12944
01-09-2010	8.08347	3.42445	0.00000	2.75629	0.00000	3.06066	0.00000
01-10-2010	8.05535	0.39514	0.00000	0.00000	0.27069	0.17257	0.15371
01-11-2010	8.00376	4.34138	0.00000	3.67981	0.00000	3.98552	0.00000
01-12-2010	7.94162	0.00000	5.72339	0.00000	6.37982	0.00000	6.12225

The table 1 simply shows the value of European call and put option at one period using BM.

5.4 Value of call and put option in BSM

In BSM estimate the value of the option (call/put) at an initial time ($t = 0$) at different strike prices that we generated by using Excel function at an expiry/mature date based on the non-dividend paying shares. The below table 2 shows the value of a call and put option using BSM (equation 7,8) at different strike prices and we used $\ln(close\ price)$ of SBI as share price at time $t = 0$.

Table 2. Value of call and put option in BSM.

Date	S_0	$N(d_1)$ Equation 7	$N(d_2)$ Equation 8	Value of call option (c)	$N(-d_1)$	$N(-d_2)$	Value of put option (p)
01-01-2010	7.62881	1.83E-19	1.26E-19	6.26E-21	1	1	3.421369
01-02-2010	7.58797	1	1	1.560599	1.11E-08	1.41E-08	5.93E-10
02-03-2010	7.63926	6.82E-61	3.45E-61	1.3E-62	1	1	7.429168
01-04-2010	7.74097	1	1	3.722722	3.71E-57	7.16E-57	7.42E-59
03-05-2010	7.72701	7.82E-05	6.62E-05	5.85E-06	0.999922	0.999934	1.314055
01-06-2010	7.74153	1	1	3.723287	3.6E-57	6.96E-57	7.21E-59
01-07-2010	7.82521	0.266254	0.252866	0.051339	0.733746	0.747134	0.262627
02-08-2010	7.92530	2.03E-24	1.33E-24	6.42E-26	1	1	4.129438
01-09-2010	8.08347	1	1	3.060659	4.28E-31	6.91E-31	1.23E-32
01-10-2010	8.05535	0.530838	0.514382	0.142253	0.469162	0.485618	0.123398
01-11-2010	8.00376	1	1	3.985517	6.75E-63	1.35E-62	1.33E-64
01-12-2010	7.94162	1.09E-43	6.14E-44	2.56E-45	1	1	6.122248

The table 2 simply shows the value of European call and put option at one period using BSM.

6. Results

Suppose that x_1, x_2 , x_3 ...... $x_n$and y_1, y_2, y_3 ...... $y_n$are two independent random samples. We set a null hypothesis

$H_{0=}$ there are no differences between the means

H_1= there are differences between the means of the two samples.

In this model, it is assumed that the variances between the two sample data are equal but unknown. The t-test statistic under H_0 is:

$$t = \frac{\bar{x} - \bar{y}}{S * \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \sim t_{n_{1+n_2-2}}$$

Where $\bar{x}$ is the mean of the sample x_1, x_2 , x_3 ... x_n and $\bar{y}$ is the mean of the sample y_1, y_2, y_3 ... y_n.

S is the standard deviation, n_1+$n_2 - 2$ is degree of freedom and n_1 and n_2 is the total elements in respective groups.

6.1. t-test for the call option and put option using BM and BSM

The hypothesis is:

H_0= there are no mean difference between BM and BSM at one period
H_1= There are mean difference between BM and BSM at one period

Table 3. The Basic statistics of options using BM and BSM.

Paired Samples Statistics

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	Put option (BM)	1.90409167	12	2.692650519	.777301251
	Put option (BSM)	1.90019192	12	2.695360837	.778083653
Pair 2	Call option (BM)	1.35776500	12	1.740456698	.502426572
	Call option (BSM)	1.35386515	12	1.743466636	.503295466

The standard deviation or variance is almost equal in call and put options using BM and BSM.

Table 4. The correlation between the options using BM and BSM.

Paired Samples Correlations

		N	Correlation	Sig.
Pair 1	Put option (BM) & Put option (BSM)	12	1.000	.000
Pair 2	Call option (BM) & Call option (BSM)	12	1.000	.000

Table 5. t-Test - Paired Two Sample for Means at 95% confidence interval between the call options.

	Call option (BM)	*Call option (BSM)*
Mean	1.357765	1.353865167
Variance	3.029189518	3.039675874
Observations	12	12
Pooled Variance	3.034432696	
Hypothesized Mean Difference	0	
Df	22	
t Stat	0.005483817	
P(T<=t) one-tail	0.497836996	
t Critical one-tail	1.717144335	
P(T<=t) two-tail	0.995673993	
t Critical two-tail	2.073873058	

Table 6. t-Test - Paired Two Sample for Means at 95% confidence interval between the put options.

	Put option (BM)	*Put option (BSM)*
Mean	1.904091667	1.900191917
Variance	7.250366818	7.264970044
Observations	12	12
Pooled Variance	7.257668431	
Hypothesized Mean Difference	0	
Df	22	
t Stat	0.003545797	
P(T<=t) one-tail	0.498601413	
t Critical one-tail	1.717144335	
P(T<=t) two-tail	0.997202826	
t Critical two-tail	2.073873058	

The correlation between the call option using BM and BSM is 1 and in case of the put option, it is also 1. It indicates that it is perfect correlated.

The 95%, confidence interval of the difference provides an estimate of the boundaries between which the true mean difference lies in 95% of all the possible random samples of 12 values. At two-tailed, we are accepting the null hypothesis because t-stat value is lying between -2.0738 and 2.0738 at 95% confidence interval in case of call option and 2.0738 and 2.0738 in case of put option and we may conclude that the mean of option values using BM and BSM at one period do not differ significantly. We can conclude that we don't have enough evidence to reject the null hypothesis.

6.2. Tukey Pair-wise Comparisons

It is used in ANOVA, in order to create the confidence interval (C.I) of all pair-wise differences between factor level means. It applies simultaneously to the set of all pair-wise comparisons.

The authors used the Turkey pair-wise comparisons in order to check whether there is any difference between a pair of groups or not. In other words, the pair of groups are statistically significant or not (the call option value using BM and BSM, put option using BM and BSM).

In these results, the table 7 shows that group contains letter A only in all factors. That letter A in all the groups indicates that there are no significant differences between the factors i.e. call option using BM and BSM, put option value using BM and BSM.

Table 7. Grouping Information Using the Tukey Method and 95% Confidence.

Factor	N	Mean	Grouping
call option value (BM)	12	1.358	A
call option value (BSM)	12	1.354	A
put option value (BM)	12	1.904	A
put option value (BSM)	12	1.900	A

Note: Means that do not share a letter are significantly different.

Table 8. Tukey Simultaneous Tests for Differences of Means.

Difference of Levels	Difference of Means	SE of Difference	95% CI	T-Value	Adjusted P-Value
call option value (BM) - call option value (BSM)	-0.004	0.711	(-1.479, 1.471)	-0.01	0.996
put option value (BM) - put option value (BSM)	-0.00	1.10	(-2.28, 2.28)	-0.00	0.997

Note: *Individual confidence level = 95.00%.*

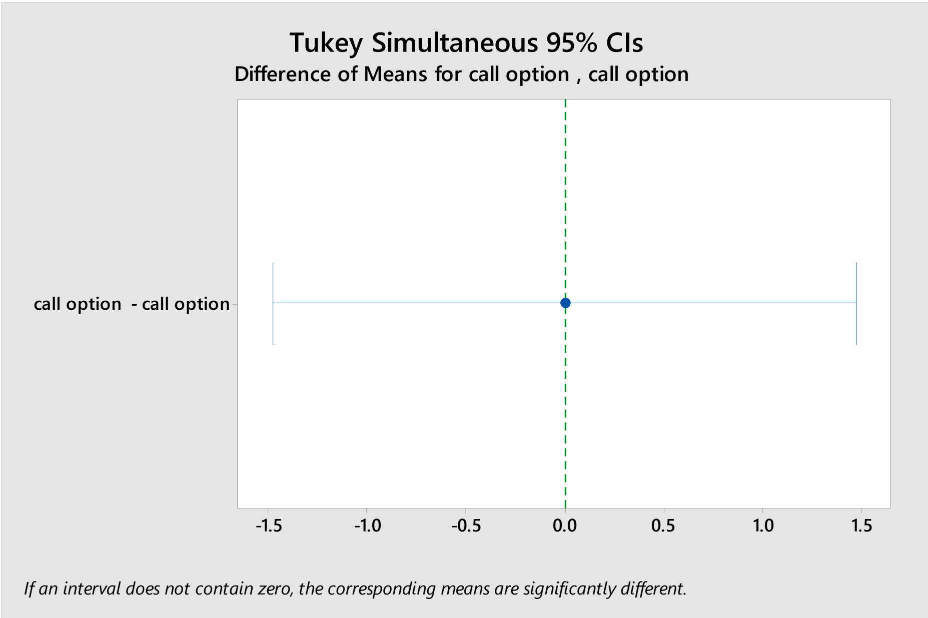

Figure 4. Tukey simultaneous 95% C.Is for call option using BM and BSM.

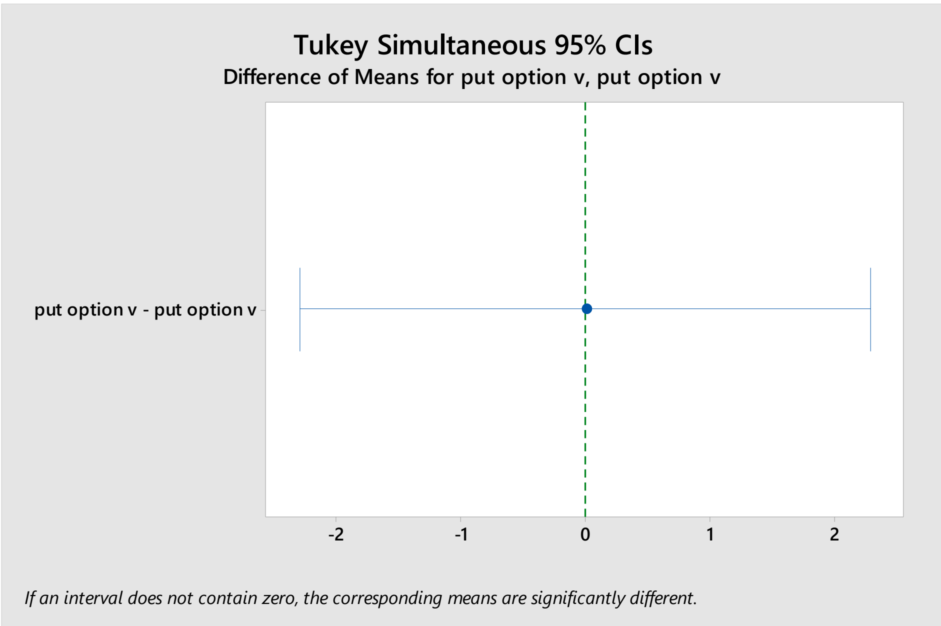

Figure 5. Tukey simultaneous 95% C.Is for put option using BM and BSM.

Table 8, shows the Tukey simultaneous C.I. The pairs, the call option value using BM and BSM and the put option value using BM and BSM have C.I. in between positive and negative which means that all the intervals include zero. The confidence interval for the means of call option (BM) - call option (BSM) and put option value (BM) – put option value (BSM) are (-1.479, 1.471) and (-2.28, 2.28) respectively. Both the pairs include zeros. If zero is not included in intervals then the means are statistically significant. But in our result both the pairs include zeros that indicate there is no significant difference between the means of the groups. The figure (4) and (5) also shows the bolded dot, which also indicates that the intervals include zero. The individual confidence level is 95%, it means that you can be 95% confident that each individual interval contains the true difference between the specific pair of groups.

7. Conclusions

The BM and BSM is an important model in a financial market in order to calculate the option pricing. In this paper, we studied how two models (BM and BSM) are approximately equal to each other at one period. The procedures we did in this paper in order to understand the difference between the two option pricing models at one period. The correlation between the call and put option using both the pricing methods at one period is perfect positive correlated. It indicated that in both the models the price of the call and put option either goes up or down. The statistical t-test showed that there are no significant differences between the call and put option using BM and BSM at 95% confidence interval. In case of Tukey method, all the factors contain only same letter '**A**' which indicates there is no significant difference between the BSM and BM. Also at 95% C.I., the intervals are involving the zero also. As per Tukey method, if zero value includes in the C.I. then there is no evidence to reject the null hypothesis. In other words, we can say that there is no statistically significant difference between the BM and BSM.

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Conflict of Interest

All authors declare no conflicts of interest in this paper.

References

Baxter M, Rennie A (1996) Financial calculus: an introduction to derivative pricing. Cambridge university press.

Blattberg RC, Gonedes NJ (1974) A comparison of the stable and student distributions as statistical models for stock prices. *J Bus* 47: 244–280.

Boyle PP (1977) Options: A monte Carlo approach. *J Financ Econ* 4: 323–338.

Carlsson C, Fullér R (2003) A fuzzy approach to real option valuation. *Fuzzy sets systems 139*: 297–312.

Cox JC, Ross SA (1976) The valuation of options for alternative stochastic processes. *J Financ Econ* 3: 145–166.

Cox JC, Ross SA, Rubinstein M (1979) Option pricing: A simplified approach. *J Financ Econ* 7: 229–263.

Feng Y, Clerence CYK (2012) Connecting Binominal and Back Scholes option pricing modes: A spreadsheet-based illustration. *Spreadsheets Education (eJSIE),* vo. 5, issue 3, article 2.

Fama EF (1965) The behaviour of stock-market prices. *J Bus* 38: 34–105.

Hull JC (2006) Options, futures, and other derivatives. Pearson Education India.

Hull J, White A (1987) The pricing of options on assets with stochastic volatilities. *J Financ* 42: 281–300.

Ingersoll JE (1976) A theoretical and empirical investigation of the dual purpose funds: An application of contingent-claims analysis. *J Financ Econ* 3: 83–123.

Liu SX, Chen Y (2009) Application of fuzzy theory to binomial option pricing model. In *Fuzzy Information and Engineering* (pp. 63-70). Springer, Berlin, Heidelberg.

Merton RC (1973) Theory of rational option pricing. *Bell J Econ manage science* 4: 141–183.

Dar AA, Anuradha N (2017) Probability Default in Black Scholes Formula: A Qualitative Study. *J Bus Econ Dev* 2: 99–106.

Black F, Scholes M (1973) The pricing of options and corporate liabilities. *J Political Econ* 81: 637–654.

Dar AA, Anuradha N (2017) One Period Binomial Model: The risk-neutral probability measure assumption and the state price deflator approach.

Mandelbrot B (1963) The variation of certain speculative prices. *J Bus* 36: 394–419.

Nargunam R, Anuradha N (2017) Market efficiency of gold exchange-traded funds in India. *Financ Innov* 3: 14.

Oduro FT (2012) The Binomial and Black-Scholes Option Pricing Models: A Pedagogical Review with VBA Implementation. *Int J Bus Inf Technology* 2.

Lazarova L, Jolevska-Tuneska B, Atanasova-Pacemska T (2014) Comparing the binomial model and the Black-Scholes model for options pricing. *Yearb Fac Computer Science* 3: 83–87.

Dar AA, Anuradha N (2017) Use of orthogonal arrays and design of experiment via Taguchi L9 method in probability of default.

Liang J, Yin H-M, Chen X, et al. (2017) On a Corporate Bond Pricing Model with Credit Rating Migration Risksand Stochastic Interest Rate.